How to Evaluate Functions at a Value Using the Rules

- Identify the independent variable in the rule of function.
- Replace the independent variable with big parentheses.
- Plug in the input that needs to be evaluated inside the big parentheses.

1. Evaluate the function $f(x) = 5x^2 - 2x + 1$ for $x = -2$. (Watch Video 1.)
2. Evaluate the function $f(x) = 5x^2 - 2$ for $x = -2$. (Watch Video 2.)
3. Evaluate the function $f(x) = 2x^2 - 4x$ for $x = b - 1$. (Watch Video 3.)
4. Evaluate the function $g(t) = 2t^2 - 2$ for $t = a + h$. (Watch Video 4.)
5. Evaluate the function $v(t) = 2t + 5$ for $t = a + h$. (Watch Video 5.)
6. Evaluate the function $g(t) = 10t - 2$ for $t = d - 2$. (Watch Video 6.)
7. Evaluate the function $f(x) = 10x + 2$ for $x = t + 2$. (Watch Video 7.)
8. Evaluate the function $f(y) = \dfrac{10y - 1}{y}$ for $y = c + 2$. (Watch Video 8.)
9. Evaluate the function $f(y) = \dfrac{5y + 2}{5y - 2}$ for $y = m + k$. (Watch Video 9.)
10. Evaluate the function $h(s) = 5 - s - \dfrac{1}{2}s^2$ for $s = j - 2$. (Watch Video 10)

Solving Equations with Multiple Parameters: PreCalculus Version)

If the desired variable only appears to power of one, then follow the following process.

> **Isolate the Variable:** First manipulate both sides so that each side clearly consists of different terms. For example, if one or both sides are quotient expressions, multiply both sides by each factor in denominator, Multiply all factors through and eliminate square roots. Add or subtract terms on both sides of the equation, make all terms on one sides contain the desirable variable and all terms on the other side do not contain that variable.
>
> **Factor the Variable:** If the desirable variable still appears to power one only, you can factor the variable on one side.
>
> **Divide:** Divide both sides by what multiplied the desirable variable.

1. Solve $P = S - Srt$ for r. (Watch Video 11.)
2. Solve $2rx + 7 = 9(r - x)$ for x. (Watch Video 12.)
3. Solve $\dfrac{1}{f} = \dfrac{2}{d_0} + \dfrac{7}{d_1}$ for f. (Watch Video 13.)
4. Solve $2ax - 7d = b(x - a)$ for x. (Watch Video 14.)
5. Solve $v = \dfrac{d + e}{1 + \frac{de}{c^2}}$ for e. (Watch Video 16.)
6. Solve $x + y = \sqrt{x^2 + y^2 + 7}$ for y. (Watch Video 17.)
7. Solve $Q_\omega = m_\omega c_\omega (T_f - T_\omega)$ for T_w. (Watch Video 18.)
8. Solve $y - y_1 = m(x - x_1)$ for x. (Watch Video 19.)
9. Solve $y - y_1 = m(x - x_1)$ for y.
10. Solve $\dfrac{x}{a} + \dfrac{y}{b} = 1$ for x. (Watch Video 21.)
11. Solve $\dfrac{1}{x} + \dfrac{1}{y} = 1$ for y. (Watch Video 22.)

Substitution Method for solving Equations. (PrecCaluclus version.)

Common Factors: Look for common factors to factor into simpler factors.

Relationship Between Exponents: Find if one of the exponents is twice or three times the other one. If there are two terms with variables and one exponent is twice the other one, expect a quadratic equation after substitution.

Substitution: The original variable to the smaller exponent becomes the New Variable.

Use one of the Types: At this point expect a quadratic or of the form $A^2 - B^2$ or $A^3 \pm B^3$. Use quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ or the difference of squares formula $A^2 - B^2 = (A - B)(A + B)$ or the sum or difference of cubes formula $A^3 \pm B^3 = (A \pm B)(A^2 \mp AB + B^2)$ to factor.

Factoring and/or Solving for the New Variable: Use each **factor** including any that may have been obtained in the first step and **SOLVE** for the New Variable.

Replace Back the Original Variable: For each value that you found, for the new variable, solve for the original variable. List all solutions with comma between them. In case no solution was possible, write NO SOLUTION. *On Gateway exam, give all exact solutions (square roots, fractions and so on.) Values such as* 2^5 *is accepted as well.*

Eliminate Extraneous Solutions: Plug back in the original equation and eliminate any extraneous solution that has been generated in the process.

1. Solve $x^{\frac{1}{3}} + 7x^{\frac{1}{6}} - 18 = 0$ for x. (Watch Video 23.)

2. Solve $(h-1)^{\frac{1}{3}} - 4(h-1)^{\frac{1}{6}} + 3 = 0$ for h.

3. Solve $18x^{\frac{1}{2}} = x + 81$ for x.

4. Solve $15z^{\frac{3}{2}} + 29z^{\frac{5}{2}} - 14z^{\frac{7}{2}} = 0$ for z, in the complex numbers domain.(Watch Video 24.)

5. Solve $z^{\frac{7}{2}} - 4z^{\frac{5}{2}} = -4z^{\frac{3}{2}}$ for z.(Watch Video 27.)

6. Solve $-10z^{\frac{1}{2}} + 41z^{\frac{3}{2}} - 21z^{\frac{5}{2}} = 0$ for z.(Watch Video 24.)

7. Solve $(2x^2 - 7)^3 - (2x^2 - 7) = 0$ for x. (Watch Video 29.) (Watch Video 29.)

8. Solve $(x+7)^3 = 8$ for x, in the complex numbers domain. (Watch Video 30.)
9. Solve $(x-11)^3 + 8 = 0$ for x, in the complex numbers domain. (Watch Video 31.)
10. Solve $2x^{\frac{1}{2}} = 18$ for x. (Watch Video 32.)

Radical Equations (PreCalculus version.)

Isolate one of the Radicals: Add or subtract terms from both sides of the equation to arrive at a equation with one radical on one side and the rest of the terms on the other side.

Both Sides to Power 2 (or whatever power that neutralizes the radical): Now that one radical is isolated, raise both side to power two. This way one of the radicals will be eliminated. Raising to power 2 for the other side of the equation MAY require a binomial calculation.

Eliminate the Next Radical if any: If the equation had more than one radical term, you may have to repeat the first and the second part.

Solve: When all radicals are eliminated, solve for the desired variable. A quadratic equation or other polynomial may be present at this stage.

Eliminate Extraneous Solutions: This stage of the work is really essential since, by squaring both side of the equation, extraneous solutions may have been produced which we need to eliminate. Plug in the solutions you found in the original equation.

1. Solve $\sqrt{x-5}+4=5$ for x. (Watch Video 35.)
2. Solve $\sqrt{5-t}=4$ for t. (Watch Video 36.)
3. Solve $c=5+\sqrt{5-c}$ for c. (Watch Video 37.)
4. Solve $r=\sqrt{r-5}+5$ for r. (Watch Video 38.)
5. Solve $2x=\sqrt{6x+28}$ for x.
6. Solve $b=\sqrt{5b-6}$ for b. (Watch Video 40.)
7. Solve $\sqrt{6-y}+\sqrt{5y+6}=6$ for y.
8. Solve $\sqrt{2x+11}-\sqrt{2x-5}=2$ for x. (Watch Video 42.)
9. Solve $\sqrt{m+7}+\sqrt{m-5}=6$ for m. (Watch Video 43.)

Substitution Method for solving Equations. (PrecCaluclus version.)

Common Factors: Look for common factors to factor into simpler factors.

Relationship Between Exponents: Find if one of the exponents is twice or three times the other one. If there are two terms with variables and one exponent is twice the other one, expect a quadratic equation after substitution.

Substitution: The original variable to the smaller exponent becomes the New Variable.

Use one of the Types: At this point expect a quadratic or of the form $A^2 - B^2$ or $A^3 \pm B^3$. Use quadratic formula $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ or the difference of squares formula $A^2 - B^2 = (A-B)(A+B)$ or the sum or difference of cubes formula $A^3 \pm B^3 = (A \pm B)(A^2 \mp AB + B^2)$ to factor.

Factoring and/or Solving for the New Variable: Use each **factor** including any that may have been obtained in the first step and **SOLVE** for the New Variable.

Replace Back the Original Variable: For each value that you found, for the new variable, solve for the original variable. List all solutions with comma between them. In case no solution was possible, write NO SOLUTION. *On Gateway exam, give all exact solutions (square roots, fractions and so on.) Values such as* 2^5 *is accepted as well.*

Eliminate Extraneous Solutions: Plug back in the original equation and eliminate any extraneous solution that has been generated in the process.

1. Solve $a^4 - 10a^2 = -21$ for a. (Watch Video 44.)
2. Solve $4x^4 = 28x^2 - 49$ for x. (Watch Video 45.)
3. Solve $2x^4 - 11x^2 - 21 = 0$ for x, in the complex numbers domain.
4. Solve $3x^4 - 23x^2 + 14 = 0$ for x. (Watch Video 47.)
5. Solve $x^4 - 9x^2 + 14 = 0$ for x. (Watch Video 48.)
6. Solve $(\dfrac{g-1}{g})^2 - 10(\dfrac{g-1}{g}) + 9 = 0$ for g. (Watch Video 49.)

7. Solve $(\frac{f+2}{f})^2 - 6(\frac{f+2}{f}) + 5 = 0$ for f. (Watch Video 50.)

8. Solve $9(\frac{x+3}{x})^2 + 6(\frac{x+3}{x}) + 1 = 0$ for x. (Watch Video 51.)

9. Solve $9(\frac{g}{g+1})^2 - 10(\frac{g}{g+1}) + 1 = 0$ for g. (Watch Video 52.)

10. Solve $25(\frac{g}{g+1})^2 - 10(\frac{g}{g+1}) + 1 = 0$ for g. (Watch Video 53.)

How to Solve Most Exponential Equations in PreCalculus

Using the Exponential Rules to simplify: If needed, use any of the rules (1) $e^x e^y = e^{x+y}$, (2) $\dfrac{e^x}{e^y} = e^{x-y}$, (3) $(e^x)^y = e^{xy}$, to create single exponential term on each side.

Setting an Equation Using the Exponents of Both Sides: Take logarithm of both side to get an equation without any exponential terms. In this step, you will use the rule $\ln(e^x) = x$.

Solve for the Variable: Solve the equation from previous step.

Extraneous Solutions: Eliminate all solutions that were generate as a result of solving the equation but are not a solution.

How to Solve Most Logarithmic Equations in PreCalculus

Using the Logarithmic Rules to Simplify: If needed, use any of the rules (1) $log_b(xy) = \log_b(x) + \log_b(y)$, (2) $\log_b\left(\dfrac{x}{y}\right) = \log_b(x) - \log_b(y)$, (3) $k\log_b(x) = \log_b\left(x^k\right)$, to create single logarithmic term on each side.

Setting an Equation Using the Exponents of Both Sides: Raise the base to power both side to get an equation without any logarithmic terms. In this step, you will use the rule $b^{\log_b(x)} = x$.

Solve for the Variable: Solve the equation from previous step.

Extraneous Solutions: Eliminate all solutions that were generate as a result of solving the equation but are not a solution.

1. Solve $2^{12t+2} = 2^{t^2+37}$ for t.(Watch Video 55.)
2. Solve $7^{10r+2} = 7^{r^2}7^{27}$ for r. (Watch Video 56.)
3. Solve $(e^{2m})^{4m} = e^{3-2m}$ for m. (Watch Video 57.)
4. Solve $(3^{3x})^x = (3^9)^x$ for x. (Watch Video 58.)
5. Solve $\ln(3x-5) = \ln(17) + \ln(2)$ for x. (Watch Video 59.)
6. Solve $\ln(x+5) - \ln(x) = 1$ for x. (Watch Video 60.)

7. Solve $\ln(x) = \ln(8) - 2\ln(x)$ for x. (Watch Video 61.)

8. Solve $\ln(4p) + \ln\left(p + \frac{7}{4}\right) = \ln(2)$ for p. (Watch Video 62.)

9. Solve $\ln(3x) + \ln\left(x - \frac{2}{3}\right) = \frac{1}{2}\ln(64)$ for x. (Watch Video 63.)

Function Operations

- $f + g$ means add the outputs.
- $f - g$ means subtract the outputs.
- $f.g$ means multiply the outputs.
- f/g means divid the outputs.
- Identify the outer and inner function. For example in $f \circ g$, f is the outer and g is the inner function.
- Write the outer and write big parentheses whenever you see the independent variable.
- Write the inner function in every parentheses.

1. Given $f(x) = x^2 + 2$ and $g(x) = \sqrt{x} - 2$, find the value of $(f - g)(9)$. (Watch Video 64.)
2. Given $f(x) = x^2 + 2$ and $g(x) = \sqrt{x} - 2$, find the value of $(f + g)(9)$. (Watch Video 65.)
3. Given $f(x) = x^2 + 2$ and $g(x) = \sqrt{x} - 2$, find the value of $(g - f)(9)$. (Watch Video 66.)
4. Given $f(x) = x^2 + 2$ and $g(x) = \sqrt{x} - 2$, find the value of $(\frac{g}{f})(a)$. (Watch Video 67.)
5. Given $f(x) = x^2 + 2$ and $g(x) = \sqrt{x} - 2$, find the value of $(gf)(x)$. (Watch Video 68.)
6. Given $f(x) = x^2 + 2$ and $g(x) = \sqrt{x} - 2$, find the value of $3g(c)$. (Watch Video 69.)
7. Given $f(x) = x^2 + 2$ and $g(x) = \sqrt{x} - 2$, find the value of $2f(1)$. (Watch Video 70.)
8. Given $f(x) = x^2 + 2$ and $g(x) = \sqrt{x} - 2$, find the value of $g(f(x))$.(Watch Video 71.)
9. Given $f(x) = x^2 + 2$ and $g(x) = \sqrt{x} - 2$, find the value of $g(f(x + y))$. (Watch Video 72.)
10. Given $f(x) = x^2 + 2$ and $g(x) = \sqrt{x} - 2$, find the value of $g(f(\sqrt{2}))$. (Watch Video 73.)
11. Given $f(x) = x^2 + 2$ and $g(x) = \sqrt{x} - 2$, find the value of $f(g(a + h))$. (Watch Video 74.)

Composition of Functions

- Identify the outer and inner function. For example in $f \circ g$, f is the outer and g is the inner function.
- Write the outer and write big parentheses whenever you see the independent variable.
- Write the inner function in every parentheses.

1. Given $g(x) = \dfrac{1}{x+3}$ and $f(x) = \sqrt{x}$, find $f(g(x))$. (Watch Video 75.)
2. Given $g(x) = \dfrac{x-1}{x+1}$ and $f(x) = x^2$, find $f(g(x))$. (Watch Video 76.)
3. Given $g(x) = \dfrac{3}{x} - x$ and $f(x) = \dfrac{x}{3} + x$, find $f(g(x))$.(Watch Video 78.)
4. Given $f(x) = \dfrac{3}{x} - x$ and $g(x) = \dfrac{x}{3} + x$, find $f(g(x))$. (Watch Video 79.)
5. Given $f(x) = x^2 + 4x - 5$ and $g(x) = x - c$, find $f(g(x))$. (Watch Video 80.)
6. Given $g(x) = 5x^2 - 2$ and $f(x) = \sqrt{x} + 1$, find $f(g(x))$. (Watch Video 81.)
7. Given $g(x) = \sqrt{x^2 - 5x}$ and $f(x) = x^2 + 1$, find $f(g(x))$. (Watch Video 82.)
8. Given $f(x) = 3x - 2$ and $g(x) = x + 1$, find $f(g(x))$. (Watch Video 83.)
9. Given $f(x) = x^2 + x^{\frac{1}{2}}$ and $g(x) = x^4$, find $f(g(x))$. (Watch Video 84.)
10. Given $f(x) = x^{\frac{1}{3}} + x^{\frac{1}{2}}$ and $g(x) = x^3$, find $f(g(x))$. (Watch Video 85.)

How to Find the Rule of Inverse Function

- Choose an output variable and set equal to the rule of the function. (For example, $y = f(x)$.)
- Solve for the input variable. (For example, x.)
- Interchange the input variable and output variable.

1. Find the inverse function of $f(x) = \dfrac{1}{5x+3}$. (Watch Video 86.)
2. Find the inverse function of $f(s) = \dfrac{-2}{5s+3}$. (Watch Video 87.)
3. Find the inverse function of $m(t) = \dfrac{3t+7}{5t}$. (Watch Video 88.)
4. Find the inverse function of $v(t) = \dfrac{2t+3}{5t-7}$. (Watch Video 89.)
5. Find the inverse function of $g(t) = \dfrac{2}{3t-5}$. (Watch Video 90.)
6. Find the inverse function of $y(x) = \dfrac{x^3-3}{x^3+7}$. (Watch Video 91.)
7. Find the inverse function of $f(x) = \sqrt{2x+3}$. (Watch Video 92.)
8. Find the inverse function of $u(t) = \dfrac{7}{\sqrt{3t}}$. (Watch Video 93.)
9. Find the inverse function of $g(y) = \sqrt{5y} + 2$. (Watch Video 94.)
10. Find the inverse function of $u(r) = 7 + \sqrt{3r-5}$. (Watch Video 95.)

Simplifying Rational Expression

Simplifying extra factor:

Factor both numerator and denominator.

Simplify the common factors.

Simplifying the Sum of Rational Expressions:

Make sure each expression is simplified. (Within the expression's domain.)

Find the least common denominator. This is going to be the new denominator.

Multiply all rational piece to make the new numerator.

After forming the new fraction, check if it can be simplified again.

1. Simplify, within its domain, as much as possible $\frac{(x^2+1)(x-1)^2}{x^4-1}$. (Watch Video 96.)
2. Simplify, within its domain, as much as possible $\frac{xy+3zy}{x^2+6xz+9z^2}$. (Watch Video 97.)
3. Simplify, within its domain, as much as possible $\frac{x^2+xy}{x^2+xy-4x-4y}$. (Watch Video 98.)
4. Simplify, within its domain, as much as possible $\frac{2x^3-4x^2+x-2}{x-2}$. (Watch Video 99.)
5. Simplify, within its domain, as much as possible $\frac{x^3+5x^2+6x}{x^3-9x}$. (Watch Video 100.)
6. Combine the rational expressions and simplify as much as possible $\frac{2}{x+3}+\frac{2}{x-3}+\frac{1}{x^2-9}$. (Watch Video 101.)
7. Combine the rational expressions and simplify, within the domain, as much as possible $\frac{-1}{x}+\frac{2}{x^2+1}+\frac{1}{x^3+x}$. (Watch Video 102.)

8. Combine the rational expressions and simplify as much as possible $\sqrt{8x-1} - \frac{x+2}{\sqrt{8x-1}}$. (Watch Video 103.)

9. Combine the rational expressions and simplify as much as possible $\frac{x}{x+y} - \frac{y}{x}$. (Watch Video 104.)

10. Combine the rational expressions and simplify as much as possible $\frac{x+h}{x+h+1} - \frac{x}{x+1}$. (Watch Video 105.)